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SOME APPLICATIONS OF WAVELETS

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ABSTRACT

The object of this paper is to prove that wavelets are adapted for the study of other functional spaces (other then $L^2(\mathbb{R})$) as $L^p(\mathbb{R})$ $(1 < p < \infty)$, $C^s(\mathbb{R})$ or $H^s(\mathbb{R})$ where s depends of the regularity of the basis. To realize this object, we prove at first some lemmas of functional analysis then we characterize some functional spaces with wavelet series. Wavelet, Vaguelet, Regular lemma, Sobolev space.

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1. INTRODUCTION

Wavelets are functions generated from one basis function by dilations and translations. The search for wavelet bases has been an active field for many years, since the beginning of the 1990'*s*. The wavelets have cancelation properties that are usually expressed in terms of vanishing polynomial moments. The combination of the two previous properties of wavelets provides a rigorous analysis of adaptative schemes for elliptic problems. Moreover, nonlinear approximation is an important concept related to adaptative approximation. The multiscale bases have been existed for a long time in search of Haar, Franklin and Littlewood-Paley. They are widely used in many scientific domains as numerical analysis or theoretical physics. Wavelet method has a great interest in signal and image processing motivated the development of Euclidean wavelets. The multiscale method is applied to the gravimetry problem, which is concerned with the determination of the earths density distribution from gravitational measurements (15).

We study in this paper other functional spaces (other then $L^2(\mathbb{R})$) as $L^p(\mathbb{R})$ $(1 < p < \infty)$, $C^s(\mathbb{R})$ or $H^s(\mathbb{R})$ where s depends of the regularity of the basis. The wavelet expansions induce isomorphisms between function and sequence spaces. It means that certain Sobolev or Besov norms of functions are equivalent to weighted sequence norms for the coefficients in their wavelet expansions which have a great flexibility and are easy to implement. The fondamental idea is to analyze some functional spaces by using a new notion called vaguelet. The main contribution offered in this paper is in applications of it results. In fact, this tpoic has many applications as numerical similation for elliptic problems or image processing, the reconstruction of the wavelet approximations of the gravitatinal potential and other important functionals of the gravitational field. Section 2 is devoted to prove regular lemmas which will be useful for the remainder of the work.

In section 3, we prove that wavelets are adapted for the study of functional spaces $L^p(\mathbb{R})$ $(1 < p < \infty)$, $C^s(\mathbb{R})$ or $H^s(\mathbb{R})$ where s depends of the regularity of the basis

2. REGULAR LEMMAS

We introduce in this section the notion of vaguelet which is very important to obtain equivalence norms on some regular functional spaces (Sobolev spaces). The following lemmas are valid in dimension n but the proofs will be done in two dimensional case to simplify notations.

Definition 2.1

A family of continuous functions $(\phi_{j,k})$, $j \in \mathbb{Z}$, $k \in \mathbb{Z}$ on $\mathbb{R}$, is called a vaguelet family if there exist constants M, α, β and C such that

$$\operatorname{supp}\phi_{j,k} \subset \{x \in \mathbb{R}/2^j x - k \in [-M, M]\}. \tag{2.1}$$

$$\left|\phi_{j,k}\right| \leq C 2^{\frac{1}{2}} (1 + |2^j x - k|)^{-1-\alpha}. \tag{2.2}$$

$$\int \phi_{j,k}(x)dx = 0. \tag{2.3}$$

$$\left|\phi_{j,k}(x) - \phi_{j,k}(y)\right| \leq C 2^{\left(\frac{1}{2}+\beta\right)j} |x - y|^{\beta}. \tag{2.4}$$

Remark 2.1: See that vaguelets have the same oscillation and estimations as wavelets but they are'nt wavelets because we cannot get them by translation and dilation.

We have the following result.

lemma 2.1: Let $(\phi_{j,k})$, $j \in \mathbb{Z}$, $k \in \mathbb{Z}$, be a family of vaguelets, then there exists a constant C such that, for every sequence $(\lambda_{j,k})$ of $l^2(\mathbb{Z}x\mathbb{Z})$, we have

$$\left\|\sum_{j\in\mathbb{Z}}\sum_{k\in\mathbb{Z}}\lambda_{j,k}\phi_{j,k}\right\|_2 \leq C\left(\sum_{j\in\mathbb{Z}}\sum_{k\in\mathbb{Z}}|\lambda_{j,k}|^2\right)^{\frac{1}{2}}. \tag{2.5}$$

Proof: Remark at first that

$$<\phi_{j,k}, \phi_{j',k'}> = 0 \text{ if } \left[\frac{k-M}{2^j}, \frac{k+M}{2^j}\right] \cap \left[\frac{k'-M'}{2^j}, \frac{k'+M'}{2^j}\right] = \varnothing.$$

Then, there exists a finite number of indexes k' such that $<\phi_{j,k'}, \phi_{j'}, k'> \neq 0$ and is increased by a constant C. We deduce that,

$$\begin{aligned}|<\phi_{j,k}, \phi_{j',k'}>| &= \left|\int \phi_{j,k}(x)\left(\phi_{j',k'}(x) - \phi_{j',k'}\left(\frac{k}{2^j}\right)\right)dx\right| \\ &\leq C2^{j/2}2^{j'\beta}\int_{\left[\frac{k-M}{2^j}, \frac{k+M}{2^j}\right]}\left|x - \frac{k}{2^j}\right|^\beta dx \\ &\leq C'2^{-(j-j')(\beta+1/2)}.\end{aligned}$$

Next, we write

$$\begin{aligned}\Sigma_{j,k\in\mathbb{Z}} 2^{(j-j')1/2}|<\phi_{j,k'}, \phi_{j',k'}>| &\leq C(\Sigma_{j<j'} 2^{(j-j')1/2}2^{-(j-j')(\beta+1/2)} \\ &+ \Sigma_{j\geq j'} 2^{(j-j')1/2}2^{-(j-j')(\beta+1/2)}2^{j'-j)} \\ &\leq C' \Sigma_{j\in Z} 2^{-|j-j'|\beta} \\ &\leq C''.\end{aligned}$$

We obtain

$$\begin{aligned}\left\|\sum_{j,k\in\mathbb{Z}}\lambda_{j,k}\phi_{j,k}\right\|_2^2 &\leq \left\{\sum_{j,k\in\mathbb{Z}}\sum_{j',k'\in\mathbb{Z}}|\lambda_{j,k}|^2 2^{(j-j')1/2}|<\phi_{j,k}, \phi_{j',k'}>|\right\}^{1/2} \\ &\times\left\{\sum_{j,k\in\mathbb{Z}}\sum_{j',k'\in\mathbb{Z}}|\lambda_{j,k}|^2 2^{(j-j')1/2}|<\phi_{j,k}, \phi_{j',k'}>|\right\}^{1/2}\end{aligned}$$

then, it gives the result.

Remark 2.2: We proved the Lemma 2.1 in one dimensional case. We have the same proof in multidimensional case.

lemma 2.2

i) If $g \in L^2(\mathbb{R}^2)$ with compact support then there exists a positive constant C such that, for every sequence $(\lambda_{k_1,k_2}) \in l^2(\mathbb{Z}^2)$, we have

$$\left\|\sum_{k_1\in\mathbb{Z}}\sum_{k_2\in\mathbb{Z}}\lambda_{k_1,k_2}g(x-k_1, y-k_2)\right\|_{L^2(\mathbb{R}^2)} \leq C\left(\sum_{k_1}\sum_{k_2}|\lambda_{k_1,k_2}|^2\right)^{1/2} \tag{2.6}$$

ii) Moreover, if $|\xi_1|^\alpha \hat{g}(\xi_1, \xi_2) \in L^2(\mathbb{R}^2)$ where $0 < \alpha < 1$, then there exists a constant positive C' such that, for every sequence $(\lambda_{k_1,k_2}) \in l^2(\mathbb{Z}^2)$, we have

$$\left\| |\xi_1|^\alpha \left(\sum_{k_1}\sum_{k_2} \lambda_{k_1,k_2} e^{-k_1\xi_1} e^{-ik_2\xi_2}\right)\hat{g}(\xi_1, \xi_2)\right\|_{L^2(\mathbb{R}^2)} \le C'\left(\sum_{k_1}\sum_{k_2}|\lambda_{k_1,k_2}|^2\right)^{1/2} \quad (2.7)$$

Proof: We assume that supp $g \subset [-L, L] \times [-L, L]$ where $L - 1$ and for $(r_1, r_2) \in \{0, ..., 4L-1\} \times \{0, ..., 4L-1\}$, we denote

$$g_{r_1,r_2} = \sum_{k_1\in\mathbb{Z}}\sum_{k_2\in\mathbb{Z}} \lambda_{r_1+4Lk_1, r_2+4Lk_2}\, g(x - r_1 - rLk_1, x - r_2 - rLk_2)$$

The distance between the supports of $g(x - r_1 - 4Lk_1, x - r_2 - 4Lk_2)$ and $g(x - r_1 - 4Ll_1, x - r_2 - 4Ll_2)$ where $(k_1, k_2) \neq (l_1, l_2)$ is bigger then $2L$. We have

$$\|g_{r_1,r_2}\|_2 = \left(\sum_{k_2}\sum_{k_2}|\lambda_{r_1+4Lk_1, r_2+4Lk_2}|^2\right)^{1/2}\|g\|_2.$$

The property (2.6) is then proved.

Now, we consider $\Omega \in L^2(\mathbb{R}^2)$ and $|\xi_1|^\alpha \hat{\Omega} \in L^2(\mathbb{R}^2)$. Then, we have the equality

$$\int\int |\xi_1|^{2\alpha} |\hat{\Omega}(\xi_1\xi_2)|^2 \, d\xi_1 d\xi_2 = \int\int\int |\Omega(x,y) - \Omega(x+h,y)|^2 \, dxdy \frac{dh}{|h|^{1+2\alpha}}$$

where

$$\frac{1}{C_\alpha} = \frac{1}{4\pi^2}\int_{-\infty}^{+\infty} |e^{ih} - 1|^2 \frac{dh}{|h|^{1+2\alpha}}$$

We write

$$\int\int |\xi_1|^\alpha |\hat{g}_{r_1,r_2}(\xi_1, \xi_2)|^2 \, d\xi_1 d\xi_2 =$$

$$C_\alpha\left(\int_{|h|<L}\int\int |g_{g_1,r_2}(x,y) - g_{r_1,r_2}(x+h,y)|^2 \, dxdy\frac{dh}{|h|^{1+2\alpha}} + \right.$$

$$\left.\int_{|h|<L}\int\int |g_{g_1,r_2}(x,y) - g_{r_1,r_2}(x+h,y)|^2 \, dxdy\frac{dh}{|h|^{1+2\alpha}}\right)$$

$$= C_\alpha(I_1 + I_2).$$

The same property for the supports of g_{r_1,r_2} used above allows

$$I_1 = \int_{|h|<L}\int\int\sum_{k_1}\sum_{k_2}|\lambda_{r_1+4Lk_1, r_2+4Lk_2}|^2 \, |g(x - r_1 - 4k_1L, y - r_2 - 4k_2L)$$

$$-g(x+h-r_1-4k_1M, y-r_2-4k_2L)\Big|^2 dxdy \frac{dh}{|h|^{1+2\alpha}}$$

$$\leq \frac{1}{C_\alpha}\left(\sum_{k_1}\sum_{k_2}\left|\lambda_{r_1+4Lk_1, r_2+4Lk_2}\right|^2\right)\left(\int\int|\xi_1|^{2\alpha}\,|\hat{g}(\xi_1,\xi_2)|^2\,d\xi_1 d\xi_2\right).$$

The second integral I_2 satisfies

$$I_2 \leq 2\int_{|h|>L}\int\int\left|g_{r_1,r_2}(x,y)\right|^2 dxdy\frac{dh}{|h|^{1+2\alpha}} = \frac{4}{\alpha L^{2\alpha}}\left\|g_{r_1,r_2}\right\|_{L^2}^2.$$

Then, the Lemma 2.2 is proved.

lemma 2.3: If g belongs to $L^2(\mathbb{R}^n)$ such that:

i) g has a compact support.

ii) $\int g(x)dx = 0$.

iii) $g \in H^\varepsilon(\mathbb{R}^n)$ for $\varepsilon > 0$.

If we denote by $g_{j,k}$ the functions $g_{j,k}(x) = 2^{jn/2}g(2^j x - k$. Then, for $j \in \mathbb{Z}$ and $k \in \mathbb{Z}^n$, there exists a positive constant C such that for every sequence $(\lambda_{j,k}) \in l^2(\mathbb{Z} \times \mathbb{Z}^n)$ and every function $f \in L^2(\mathbb{R}^n)$ we have:

$$\|\Sigma_{j\in\mathbb{Z}}\,\Sigma_{k\in\mathbb{Z}}\,n\,\lambda_{j,k}\,g_{j,k}\|_{L^2(\mathbb{R}^n)} \leq C(\Sigma_{j\in\mathbb{Z}}\,\Sigma_{k\in\mathbb{Z}}\,n|_{j,k}|^2)^{1/2}. \quad (2.8)$$

and

$$(\Sigma_{j\in\mathbb{Z}}\,\Sigma_{k\in\mathbb{Z}}\,n|\langle g_{j,k}\rangle|^2)^{1/2} \leq C\|f\|_{L^2(\mathbb{R}^n)}. \quad (2.9)$$

Proof:

We prove first this Lemma for $n = 2$. We denote by

$$\Omega_j = \sum_{k_1\in\mathbb{Z}}\sum_{k_2\in\mathbb{Z}}\lambda_{j,k_1,k_2}\,2^j g_i(2^j x - k_1, 2^j y - k_2).$$

We consider $0 < \varepsilon < 1$. Then, there exists a positive constant C such that

$$\left\|\left(|\xi|^\varepsilon + |\eta|^\varepsilon\right)\hat{\Omega}_j(\xi,\eta)\right\|_{L^2(\mathbb{R}^2)} \leq C2^{j\varepsilon}\left(\sum_{k_1\in\mathbb{Z}}\sum_{k_2\in\mathbb{Z}}\left|\lambda_{j,k_1,k_2}\right|^2\right)^{1/2}.$$

We have

$$\int\int g dxdy = 0.$$

Then, we write

$$g = \frac{\partial}{\partial x_1}g_1 + \frac{\partial}{\partial x_2}g_2,$$

where g_i, $i = 1, 2$, belongs to $L^2(\mathbb{R}^2)$, g_i has compact support and $\xi_i\hat{g}_i(\xi_1,\xi_2) \in L^2(\mathbb{R}^2)$. We conclude from Lemma 2.2 applied to g_i, $i = 1, 2$ and $\alpha = 1 - \varepsilon$, that if we denote by

$$\Omega_{i,j}=\sum_{k_1\in\mathbb{Z}}\sum_{k_2\in\mathbb{Z}}\lambda_{j,k_1,k_2}2^j g_i(2^j x-k_1,2^j y-k_2),$$

we have

$$\left\|(|\xi|+|\eta|)^{-\varepsilon}\hat{\Omega}_j(\xi,\eta)\right\|_2\le 2^{-j}\left(\left\||\xi|^{1-\xi}\hat{\Omega}_{1,j}(\xi,\eta)\right\|_2+\left\||\eta|^{1-\varepsilon}\hat{\Omega}_{2,j}(\xi,\eta\right\|_2\right),$$

$$\le C2^{-j}2^{j(1-\varepsilon)}\left(\sum_{k_1}\sum_{k_2}\left|\lambda_{j,k_1,k_2}\right|^2\right)^{1/2}=C2^{-j\varepsilon}\left(\sum_{k_1}\sum_{k_2}\left|\lambda_{j,k_1,k_2}\right|^2\right)^{1/2}.$$

where C is a positive constant. Then, we obtain, for $j\ge l$:

$$\left|\langle\Omega_1|\Omega_l\rangle\right|=\frac{1}{4\pi^2}\left\|(|\xi|+|\eta|)^{-\varepsilon}\hat{\Omega}_j\right\|_2\left\|(|\xi|+|\eta|)^{\varepsilon}\hat{\Omega}_l\right\|_2,$$

$$\le C2^{-\varepsilon|j-l|}\left(\sum_{k_1}\sum_{k_2}\left|\lambda_{j,k_1,k_2}\right|^2\right)^{1/2}\left(\sum_{k_1}\sum_{k_2}\left|\lambda_{l,k_1,k_2}\right|^2\right)^{1/2}.$$

Thus there exists a positive constant C' such that

$$\left\|\sum_{j\in\mathbb{Z}}\Omega_j\right\|_2\le C'\left(\sum_j\sum_{k_1}\sum_{k_2}\left|\lambda_{j,k_1,k_2}\right|^2\right)^{1/2}.$$

We have the same proof for every $n\ge 2$. Then the Lemma 2.3 is proved.

3. ANALYZE BY WAVELETS

We recall that the decomposition of a function f in a series of wavelets is given by

$$f(x)=\Sigma_{j,k\in\mathbb{Z}}(f,\psi_{j,k})\psi_{j,k}(x)=\Sigma_{j,k\in\mathbb{Z}}c_{j,k}\psi_{j,k}(x)$$

We characterize in this section the spaces $L^p(\mathbb{R})$, $C^s(\mathbb{R})$ and $H^s(\mathbb{R})$. We have the first result.

Theorem 3.1: Let $V_j(\mathbb{R})$ be a regular multiresolution analysis and $(\psi_{j,k})_{j,k\in\mathbb{Z}}$ the associated wavelet basis. Then, for $1<p<\infty$, wa have

$$\|f\|_p\sim\left\|\left(\sum_{j,k\in\mathbb{Z}}\left|C_{j,k}\right|^2\left|\psi_{j,k}(x)\right|^2\right)^{\frac{1}{2}}\right\|_p$$

The proof can be deduced from lemmas 2.2 and 2.3.

Definition 3.1

i) A function f belongs to $C^s_{x_o}$ if there exists a polynomial P of degree lower or equal to entire party of s such that

$$f(x) = P(x - x_o) + 0\,(|x - x_o|^s).$$

ii) $f \in C^s(\mathbb{R})$ if $f \in C^s_{x_o}$ for every $x_o \in \mathbb{R}$ and if $0(x)$ is uniform in x_o.

Theorem 3.2: We assume that $s \notin \mathbb{N}$ and that the wavelet ψ is C^{s+1} and has its first moments null. If we denote:

$$C_{j,k} = \int f(x)\psi_{j,k}(x)dx,$$

then, we have the following equivalence:

$$f \in C^s(\mathbb{R}) \Leftrightarrow \left|C_{j,k}\right| \le C2^{-\left(\frac{1}{2}+s\right)j}.$$

Proof: If $f \in C^s$, we have

$$|C_{j,k}| = |\int f(x)\psi_{j,k}(x)dx| = \left|\int f(x) - P\left(x - \frac{k}{2^j}\right)\right)\psi_{j,k}(x)dx\right|$$

(because ψ has some moments null and is fast decreasing)

then

$$\left|C_{j,k}\right| \le C\int\left|x - k2^{-j}\right|^s \frac{2^{\frac{1}{2}}dx}{\left(1+2^j\left|x-k2^{-j}\right|\right)^N}$$

$$\le C2^{-\left(\frac{1}{2}+s\right)j}\int\frac{|y|^s\,ds}{(1+|y|)^N} \le C'2^{-\left(\frac{1}{2}+s\right)j}.$$

Inversely, we assume that

$$\left|C_{j,k}\right| \le C2^{-\left(\frac{1}{2}+s\right)j}$$

We denote

$$Q_j(f) = \Sigma_k C_{j,k}\,\psi_{j,k}.$$

We have

$$\|Q_j(f)\|_\infty \le C2\text{–}^{sj}$$

and

$$\|\partial^\alpha(Q_j(f))\|_\infty \le C2^{(\alpha-s)j}$$

(due to the localization property of ψ).

Let $x_o \in \mathbb{R}$. We denote

$$P_j(x - x_0) = \sum_{j<\alpha}\frac{(x-x_o)^\alpha}{\alpha!}Q_j^{(\alpha)}(f)(x_o)$$

and

$$P(x - x_o) = \Sigma_{j\geq o} P_j(x - x_o).$$

This series converges. Then, if j_o is such that

$$2^{-j_o} \leq |x - x_o| < 2\ 2^{-j_o},$$

we have

$$| f(x) - P(x - x_o)|\ \Sigma_{j\leq j_o} |Q_j(f)(x) - P_j(x - x_o)| + \Sigma_{j>j_o}|Q_j(f)(x) - P_j(x - x_o)|.$$

The first sum is increased by

$$\Sigma_{j\leq j_o}|x - x_o|^{[s]+1}\sup_{|\alpha|=[s]+1} \|\partial^\alpha Q_j(f)\|_\infty \leq C\ \Sigma_{j\leq j_o} |x - x_o|^{[s+1]}\ 2^{([s]-s+1)j}$$

The second sum is increased by

$$C\sum_{j>j_o}\left(\left\|Q_j(f)\right\|_\infty + \sum_{a<|s|}\left|x - x_o\right|^\alpha \left\|Q_j^{(\alpha)}(f)\right\|_\infty\right)$$

$$\leq C\sum_{j>j_o}\left(2^{-sj} + \sum_{\alpha<|s|}\left|x - x_0\right|^\alpha 2^{-(j-\alpha)}\right)$$

$$\leq C\ |x - x_o|^\alpha.$$

Proposition 3.1: If $f \in H^s(\mathbb{R})$ $(s > 0)$ then we have the following inequality:

$$\left\| f - P_j(f)\right\|_{L^2} \leq C\ 2^{-JS} \left\| f\right\|_{H^s}.$$

Proof: If $f \in H^s(\mathbb{R})$, we have the inequality

$$\sum_{j,k}\left|C_{j,k}\right|^2 (1+2^{2j})^s \leq C\left\| f\right\|_{H^s}^2.$$

Then

$$\left\| f - P_J(f)\right\|_{L^2}^2 = \sum_{j\geq J}\left|C_{j,k}\right|^2$$

$$\leq 2^{-2Js}\sum_{j\geq J}(1+2^{2j}\left|C_{j,k}\right|^2$$

$$\leq C\ 2^{-2Js}\left\| f\right\|_{H^s}^2.$$

The proposition is then proved.

Proposition 3.2: $f \in H^s(\mathbb{R})$ $(s > 0)$ if and only if we have

$$\sum_{j,k}\left|C_{j,k}\right|^2 (1+2^{2j})^s < \infty$$

Proof: The first sense results from Proposition 3.1 and the second sense results from the Bernstein estimate.

4. CONCLUSION

We studied in this paper other functional spaces (other then $L^2(\mathbb{R})$) as $L^p(\mathbb{R})$ $(1 < p < \infty)$, $C^s(\mathbb{R})$ or $H^s(\mathbb{R})$ where s depends of the regularity of the basis. We proved the wavelet expansions induce isomorphisms between function and sequence spaces. It means that norms of some functions spaces are equivalent to weighted sequence norms for the coefficients in their wavelet expansions.

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